RESEARCH PAPER

One Teacher's Pedagogical Actions in Eliciting and Developing Mathematical Reasoning Through a Contextually Relevant Task

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In this paper, we report on the pedagogical actions of one teacher in eliciting and developing students' mathematical reasoning during one mathematics lesson. The findings illustrate that through the careful design and planning of a contextually relevant task (the construction of a manu tukutuku), and the implementation of specific teacher actions, a group of marginalised students were provided access to exploring the concept of equivalence.

In this paper, we aim to illustrate pedagogical actions teachers may take to engage marginalised students in learning important mathematical concepts. To support students in learning mathematics, teachers need to enact specific instructional actions to ensure all students can access the content. One of these actions is to provide multiple opportunities for engagement in collaborative mathematical discussions characterised by students asking questions, making conjectures, justifying, and generalising, not only with their own thinking, but also with the thinking of their peers (Selling, 2016). Some teachers, however, encounter challenges in creating the kinds of collaborative learning environments required for students to participate in productive discourse. One barrier could be the teachers' strong beliefs that mathematics is best learned through students reproducing teacher directed procedures with solution focused outcomes. Another challenge is that some teachers hold deficit views about certain students being more or less capable of learning mathematics than others. One way to mitigate these challenges is for teachers to provide students with many opportunities to engage in learning mathematics collaboratively through contextually relevant tasks.

In this paper, we report on findings from a small inquiry, where one teacher used one contextually relevant task to engage marginalised students to reason with mathematical concepts. The findings also highlight that when the teacher enacted several certain instructional actions, the students were supported to reason collectively and engage in mathematical discourse. The specific research question explored in this paper is:

> *How can a contextually relevant task engage marginalised students to reason mathematically?*

Literature Review

One teacher action that supports productive engagement in mathematical reasoning is providing students with an appropriately challenging task. These kinds of tasks need to be open-ended, high-level, focused on important mathematical concepts, with various entry and exit points allowing for student creativity and multiple solution pathways. When teachers design tasks in these ways, they are providing opportunities for students to think, reason, and problem solve in cognitively demanding ways (Smith & Stein, 1998). Solving routine tasks using memorised procedures, without connections to mathematical concepts does not allow for new learning to occur, rather, completing the task is simply practicing a learned procedure or formula. When these kinds of classroom practices take place, there is little opportunity for students to explore or deepen their understanding of important mathematical concepts. Askew (2012) suggested that mathematics tasks should prompt students to engage in mathematical reasoning as opposed to being led by direct-teacher instruction. Such a pedagogical approach allows students to become active participants in the learning process, co-constructing new

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knowledge through improvising solutions, rather than relying on teacher direction. Furthermore, both Selling (2016) and Mueller et al. (2014) agreed that challenging tasks provide teachers with affordances to facilitate meaningful discussions with students, that elicit their thinking and ideas. Marginalised students can benefit from learning mathematics in this way as when they experience success in finding solutions to problems in multiple-accepted ways, and are not always being shown what to do, they can begin to see themselves as knowers and doers of mathematics (Sullivan et al., 2020).

When first presented with a high-level challenging task, without any direct teacher direction on which strategy or solution pathway to use, students may initially feel overwhelmed or confused, anxious, or resist attempting the task altogether (Selling, 2016). While high-level cognitive demanding tasks can initially appear daunting to some students, Smith and Stein (1998) emphasised that low level tasks do not provide pathways to high-level mathematical reasoning. Askew (2012) described how the gap between mathematical content and individual understanding can be bridged using a context that is meaningful to children. He suggested that students may not generate solutions to a problem if they do not see the relevance of the problem to real-life experiences. Bills and Hunter (2015) found that the use of relevant cultural contexts can provide marginalised students with support for the development of conceptual understanding. Sullivan et al. (2020) emphasised the benefit for teachers to design a range of tasks sequencing mathematical concepts that can be explored and consolidated. Therefore, it is imperative that teachers focus on specific mathematical concepts and learning goals their students need to explore. High-level tasks should also provide opportunities for students to make conjectures, reason about, and justify their mathematical thinking (Hunter, 2014). For students to engage in mathematical discourse characterised by conjectures, explanations, and justification, they need to be explicitly supported to do so.

Students can be supported to engage in productive mathematical discourse by specific teacher action. One pedagogical approach is the use of *talk moves* (Chapin et al., 2003). Talk moves are teacher-initiated requests for students to add on, repeat, or revoice ideas presented by their peers. Talk moves also support the development of mathematical argumentation as students can be asked to agree or disagree with the mathematical content of their peers. Threaded through these moves is the intentional use of wait time, an action which provides students with time to construct their responses, or to clarify and reason with ideas being presented by their peers. Another instructional action teachers can utilise to establish purposeful discourse is to select and sequence group explanations after students have worked collectively on mathematical activity. Selecting and sequencing explanations offers affordances for mathematical ideas and concepts to be explored in greater depth (Stein et al., 2008; Sullivan et al., 2020). Furthermore, when mathematical explanations are purposefully sequenced, students are able to contribute to the construction of mathematical understanding in ways that draw on their strengths. Finally, it is essential that the teacher connects students' responses and ideas to important mathematical concepts. Selling (2016) described this as a *reprisal* move-an action where the teacher goes beyond simply accepting the explanations groups of students have presented. Rather, the teacher makes explicitly draws a connection to the mathematical concepts inherent in the groups' solution strategies and explanations. This action ensures access to deeper mathematical understanding for all students.

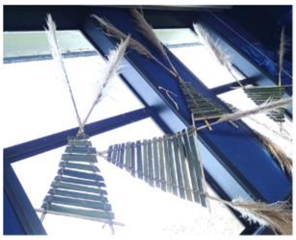
Research Methods

The present qualitative inquiry was grounded in a sociocultural perspective and was undertaken in one mathematics class in a regional primary school in Aotearoa, New Zealand. The students were aged 11–13 years old. The academic achievements in mathematics for these students highlighted that many of them were not yet achieving at the expected level (Level Four) of the New Zealand Curriculum, as evidenced by performance in standardised school assessments. The teacher was a participant in a nation-wide professional learning and development project focused on equity and inclusivity in mathematics education.

Qualitative data were collected from field notes, classroom artifacts, videorecording, and transcripts of student and teacher dialogue. Analyses comprised of multiple reviews of the recorded footage and transcribed dialogue, and thematic identification. Classroom episodes, where opportunities had been provided for students to engage in mathematical reasoning generated, which were then used for coding.

The task designed for this lesson formed part of a larger unit on Fractions. The students had some prior experience ordering fractions and their relative decimals and percentages. The big mathematical idea in this task was equivalence. The task provided opportunities for students to explore how fractions with different denominators could be subtracted from a whole. Prior to the lesson commencing, the teacher stated that she was aware that while some students could use a learned procedure to convert to equivalent fraction, being able to reason with why the procedure worked was the intended focus of the lesson.

The context of the task had been written by the teacher after noticing how the students had worked together to create manu tukutuku (traditional Māori kites) during Matariki (Māori New Year) celebrations. She had reflected on conversations among students about how they could get the most pieces out of their lengths of harakeke (flax). This task provided connections to both a lived experience of the students, and a cultural connection for Māori students in the class, thus serving as an entry point for all students (Bills & Hunter, 2015; Sullivan et al., 2020).



Dion and Jayden were cutting harakeke for their manu tukutuku. They discovered that if they cut carefully, they could cut one long piece of harakeke into 4 different sized pieces, that fit on their manu tukutuku.

Piece one is ¼ of the harakeke, Piece 2 is 2/5 of the harakeke, piece 3 is 3/10 of the harakeke.

What fraction of the harakeke is piece 4?

Figure 1. Culturally responsive task used in lesson.

While planning this task, the teacher had anticipated several possible strategies the students might attempt as they solved the task. This included drawing four different rectangles or circles and comparing each sized piece and trying to work out how much was left, or, drawing one rectangle or circle and trying to show each fraction on it, or being able to convert fifths into tenths and then possibly twentieths, or the use of decimals and percentages to convert all four fractions to determine how much of the harakeke was left for piece four.

The teacher launched the task by initially inviting the students to discuss what they had experienced while cutting their harakeke into the various sized pieces. The students shared how they had measured the pieces and cut carefully so that no harakeke was wasted, and how they could get as many long and short pieces out of one length. The teacher then prompted them to describe how they had to work together to build their manu tukutuku. These ways were then connected back to the social norm of working together to solve the task. These actions have been identified by Hunter and Civil (2021) to show the students the connections and value of prior skills to the current mathematical activity.

Findings and Discussion

The findings and discussion are presented chronologically. The specific instructional actions the teacher used are emphasised. Key themes appear as headings for each section. The explicit outcomes and opportunities for deepening mathematical reasoning are illustrated through the students' responses and dialogue.

Teacher Noticing and Responding to Students' Mathematical Reasoning

As the groups worked on solving the task, the teacher monitored the groups carefully by observing closely and taking care to notice the different strategies groups were using to solve the problem. This specific instructional action has been highlighted by several researchers (e.g., Jacobs & Empson, 2016; Smith & Stein, 1998) as an effective way of responding in the moment to students mathematical reasoning.

Initially, the teacher noticed that most groups explored the problem with representations of the fractional pieces of the harakeke. One group began by drawing three circles and attempted to divide the circles into different fractional regions. For example, dividing one circle into fifths, and another into tenths. After heading into difficulty to divide the circles accurately into the fractional sections, the students altered their approach. One student reminded the group members that the task concerned the length of harakeke (flax) and suggested it might be easier to use a rectangle to represent the harakeke. The teacher noticed that another group represented their reasoning by stacking several rectangles to illustrate all fractional pieces at the same time.

Whilst monitoring the small group work, the teacher listened carefully to students' explanations and questioning. This was an important pedagogical action, as listening to the developing mathematical discourse meant the teacher was able to begin selecting which groups would share their thinking, and the mathematical reasoning could be purposefully sequenced to develop student understanding.

Selecting and Sequencing Students' Solutions

All teacher anticipated responses were evident across the groups. Careful attention was also paid to claims students were making about the fractions such as "one-quarter is the biggest piece" and "one-quarter is two and a half one-tenths". Thought then went into how these statements could be shared with the larger group as points to argue and how the representations students had drawn could prove or disprove these claims. The sequencing of the group explanation aimed deliberately to support the explanations building on one another in increasing sophistication. This specific instructional action has been documented to support students developing conceptual understanding of important mathematics ideas (Stein et al. 2008).

The first group to share was chosen as they had not found the remaining fraction, however, they had made a mathematical claim about the need to convert all the fractions to equivalent fractions to be able to subtract them. Further, they stated that they did not think that tenths could be converted to quarters. An extract from this discussion can be seen below:

> Student 1: We decided to start with three-tenths, and we have tried to find the equivalent of one and one-tenth
> Teacher: Because?
> Student 2: Because they have to be the same fraction and one-quarter is bigger than one-tenth because this piece is bigger than that piece (points to diagram of two rectangles drawn on the board) and you cannot split it
> Teacher: What do you mean by 'you cannot split it'?
> Student 1: You cannot evenly split tenths into quarters
> Teacher: So, you are making a claim that you cannot evenly divide tenths into quarters?
> Student 2: Yes, because half of 10 is five, and then it is not even

The teacher used sustained questioning and then a reprisal move (Selling, 2016) to clarify thinking and elicit a deeper explanation from the wider group. These moves acknowledge the small group explaining as valid contributors to the developing mathematical thinking, in spite of not yet having solved the task. The students continued building on their explanation. The excerpt below illustrates the deepening dialogue:

> Teacher: Can you show on your diagram what this looks like that shows what you mean by its not even?
>
> The group draw one rectangle divided into tenths and dissect the rectangle showing half, labelling each half as one-fifth
>
> Student 1: See, you cannot halve five evenly

The group stood silently waiting. The teacher then extended an invitation to all students asking if anyone would like to add to what had been said.

> Student 3 (On mat): Well, it is still even because half of 5 is 2.5

The teacher asked the small group to consider how this claim could be represented or proved. Together, they draw lines on the rectangle to show quarters. Once the group had completed their drawing, she asked them think about a mathematical statement they could make about the number of tenths that were equivalent to one-quarter. Two students offered the following reasons:

> Student 1: Oh wait! It is two and a half
>
> Student 4 (On mat): Its it is two and a half one-tenths. And the other bit is three-tenths so that is five and a half one-tenths altogether.

In this episode, the teacher had initiated a group discussion where all ideas were considered. Specifically, she utilised wait-time and adding on, both of which are important pedagogical actions supporting the development of productive discourse, as highlighted in the work of Chapin et al. (2003).

The teacher continued developing the collaborative discussion by asking the students if they agreed with the statement that two and a half one-tenths was the same size as one-quarter. Several students agreed while others looked confused. The teacher waited and then prompted the students to turn and talk with each other to make sense of what had been stated. Some students used both explanations and written representations (diagrams) to communicate their reasoning with other members of their groups. The teacher then asked two students who had initially appeared confused to share their explanations with the larger group. She asked two other students to add on to the previous claim that they had cut five and a half one-tenths by adding one-quarter and three-tenths. The students were able to do this by pointing to the diagram the group had drawn on the board. Now satisfied that there was an improved understanding of the relationship between one-quarter and two and a half tenths, the teacher offered a reprisal move (Selling, 2016) to highlight the mathematical reasoning that has just emerged.

> Teacher: What you have just explained and shown the group is that by dividing the harakeke into tenths you can show where the cuts were made to get the pieces that were one quarter and three-tenths long.

The teacher drew attention to the clear explanation the group had made. Continuing, she asked the second group to build on the explanation and explain how they had converted two-fifths to four-tenths. This group of students drew two stacked rectangles, one divided into fifths, and the other into tenths. Some students became confused as to why there were now what appeared to be two separate pieces of harakeke. The teacher then prompted one of the students to ask the group explaining why they had represented their thinking in that way:

> Student 1 (On mat): Why have you got two pieces of harakeke? There was only one
>
> Student 2 (From the group explaining): There is only one, this is just to show how tenths and fifths are

> the same
> Teacher: Why do you think it is important to know how many tenths are the same as one-fifth?
> Student 3 (From the group explaining): Because then we can work out how big each piece was that we cut
> Teacher: What are we trying to find out in this task?
> Student 4 (On mat): Oh, how big the left-over piece was
> Teacher: So, can you now show on one rectangle where the cuts are, if we know that one-quarter is the same as two and a half tenths, two-fifths is equivalent to four-tenths, and we also have a piece that is three-tenths?

The group explaining then drew a new rectangle and divided it into tenths. They shaded each unit and were left with what they described as half of one-tenth. The teacher then asked the students to think about what fraction the left-over piece was, asking whether half of one-tenth was the best way to describe it. One student responded stating that half of one-tenth was not a real fraction. Another group of students explained that they had recognized that quarters, fifths, and tenths could all evenly divide into twentieths as seen in the following extract:

> One-quarter is the same as five-twentieths, and three-tenths is six-twentieths, and two-fifths is the same as eight-twentieths.
>
> When we add these together, we get nineteen-twentieths, with one-twentieth remaining

Satisfied with the development of students' reasoning, the teacher proceeded with connecting all the students' ideas to important mathematical concepts inherent in the task.

Connecting Students' Reasoning

To connect these ideas the teacher drew another rectangle on the board. This time she divided it up into twentieths. She asked the students to think about where halfway was. One student explained it was "at 10 because 10 is half of 20". She then asked where one-quarter would be, and another student explained that one-quarter would "be at five because 20 divided by four is five." The teacher then counted the twentieths out in groups of five to illustrate this. She then turned to the final groups' explanation and asked the students to think about where the fifths were. They explained that 20 divided by five was four, so, fifths would be the same as four twentieths. The teacher drew lines to highlight where the fifths were. Finally, she asked where the tenths were. At this point, the teacher was now certain that the students understood why twentieths could be used to solve the problem. She then asked all the students to shade in the pieces of harakeke using twentieths. The teacher actions in connecting the different groups ideas supported students to understand that there was now only one-twentieth remaining. The teacher then extended student reasoning to generalizing with how many twentieths were equivalent to four-fifth, three-quarters, and seven-tenths-which many of the students were able to do.

Conclusions

The findings of this investigation have illustrated how using a contextually relevant task, and specific instructional actions can support underachieving students to learn important mathematical concepts. Underpinning these actions was evidence of the teacher utilising an asset-based approach to teaching and learning mathematics. Teaching mathematics from a strength-based approach involved the teacher holding high expectations that all students were capable of learning mathematics, and countering deficit theorising about others.

Analyses of the data highlighted that when all students, particularly underachieving students were supported to learn mathematics though collaborative engagement in challenging tasks, they were provided affordances to explore important mathematics concepts. These affordances included opportunities to explain their mathematical reasoning, reason with their peers' explanations, ask questions for clarification, and justify thinking. The findings from this inquiry also illustrated that one specific way students could be supported to participate effectively in collaborative mathematical activity was through purposeful and consistent development of social norms. Evidence suggested that when effective ways of working together had been established, students developed confidence to discuss their ideas publicly, even when solutions were incomplete or showed obvious errors or misconceptions. Parallel to developing effective collaborative norms, was the importance of planning tasks that were meaningful to students.

In this investigation, effective planning involved thoughtful consideration of the cultural identities of the students and drawing on these to connect to important mathematical concepts in contextually relevant ways-in this instance, the fractional reasoning required for making manu tukutuku. Planning actions also included teacher anticipation of probable student responses and the identification of possible misconceptions or common errors. Planning mathematical tasks in this way ensured the teacher had clear reference points for monitoring student collaboration. Monitoring is an instructional action characterised by the teacher noticing and in-the-moment responding to students working together on mathematical activity. In-the-moment noticing and responding in this inquiry resulted in students clarifying mathematical explanations, asking questions, and representing their reasoning in different ways for conceptual understanding. Wider student discussions were also made possible when the teacher extended invitations for all students to contribute to the mathematical reasoning.

Group solution pathways were also sequenced in a way that afforded deeper mathematical thinking through shared discussion about the different mathematical ideas. This action meant that students' collective reasoning could be built on in increasing sophistication. The findings presented in this paper highlighted that when clear and explicit connections were drawn between students' co-constructed mathematical reasoning, the students had access to many ways to think about the mathematical concept of equivalence within a contextually relevant task. The implications of these findings demonstrate the benefits for all students when teachers adopt pedagogical approaches to teaching mathematics that encompass equity and inclusion. Teaching and learning mathematics with equity is particularly important in New Zealand, as for many students, mathematics is an unyielding gatekeeper where access is often only provided for those who exhibit speed and accuracy. When teachers purposefully plan mathematical activity that connects authentically to their students' experiences and utilise specific instructional actions that provide opportunities for creative exploration and reasoning, conceptual mathematics learning becomes accessible to all.

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